MA 161 - Lesson 7 (2.5)

Today: Limits at infinity
Horizontal Asymptote

Office Hours: Monday, Wednesday, Friday 2:45pm - 4:15pm

Announcements: Exam 1 on Thursday 09/18, 8pm - 9pm
↳ Study Guide, instructions, Seating Chart on Brightspace.

HW7 Due: Thursday 09/09

Quiz 4 (Lesson 5,6): Thursday 09/09

Review example: Determine the vertical asymptotes of

$$f(x) = \frac{(x-1)^2(x+3)}{(x-1)(x+3)^2(x-7)}$$

possible when denominator $= 0$

$\rightsquigarrow$ $x=1$, $x=-3$, $x=7$

$x \neq 1$
$x \neq -3$
$x \neq 7$

$$f(x) = \frac{(x-1)}{(x+3)(x-7)}$$

$$\boxed{\lim_{x \to 1} f(x) = 0}$$

$\rightsquigarrow$ $x = 1$ $\rightsquigarrow$ NOT a V.A

When close to -3 or 7 $\rightsquigarrow$ $\dfrac{\text{constant Number}}{\text{Very Small Number}}$ = Very Large Number

$x=-3$, $x=7$ are V.A

Evaluate $\lim_{x \to a} \frac{1}{x}$

Last class: $a = 0$ ~> $\lim_{x \to 0^+} \frac{1}{x} = \frac{1}{\text{small } +ve} = +\infty$

$\lim_{x \to 0^-} \frac{1}{x} = \frac{1}{\text{small } -ve} = -\infty$

Today: take a Larger and Larger

$a \neq 0$ $\lim_{x \to a} \frac{1}{x} = \frac{1}{a}$

$\lim_{x \to 10} \frac{1}{x} = \frac{1}{10} = 0.1$

$\lim_{x \to 100} \frac{1}{x} = \frac{1}{100} = 0.01$

$\lim_{x \to 100000} \frac{1}{x} = \frac{1}{100000} = 0.00001$

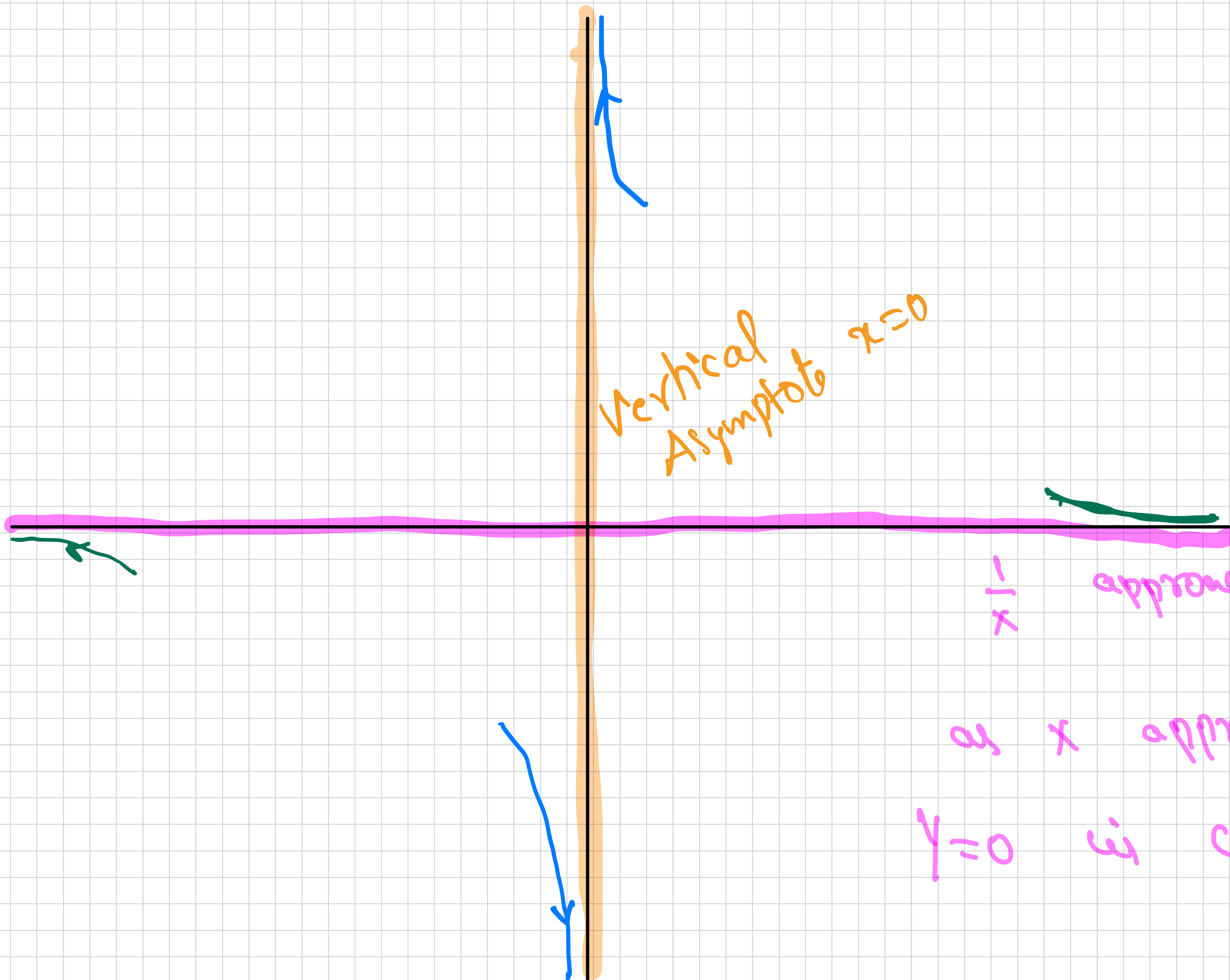
$\lim_{x \to -100000} \frac{1}{x} = \frac{-1}{100000} = -0.00001$

Notations:

$\lim_{x \to +\infty} f(x)$ } $f(x)$ as x goes to Larger & Larger +ve Numbers

$\lim_{x \to -\infty} f(x)$ } $f(x)$ value as x goes to Larger & Larger -ve values.

$$\lim_{x \to +\infty} \frac{1}{x} = 0 \quad \text{and} \quad \lim_{x \to -\infty} \frac{1}{x} = 0$$



$\frac{1}{x}$ approach this Horizontal Line $y=0$

as x approach $+\infty$ & $-\infty$

$Y=0$ is called Horizontal Asymptote.

finite number

$y = L$ is a Horizontal Asymptoto to $y = f(x)$

if $\lim_{x \to +\infty} f(x) = L$ or $\lim_{x \to -\infty} f(x) = L$

in General Any function $f(x)$ has 0, 1 or 2 H.As.

eg! Let $c \neq 0$ be a constant, $n > 0$

$\lim_{x \to \infty} \frac{c}{x^n}$ and $\lim_{x \to -\infty} \frac{c}{x^n}$

take Larger & Larger +ve values

eg! $c = 7$, $n = 2$

$x = 10 \rightsquigarrow \frac{c}{x^n} = \frac{7}{10^2} = \frac{7}{100} = 0.07$

$x = 100 \rightsquigarrow \frac{c}{x^n} = \frac{7}{100^2}$

$x = 100000 \rightarrow \frac{c}{x^n} = \frac{7}{(100000)^2}$

Getting Larger

getting closer to 0

$\lim_{x \to \infty} \frac{c}{x^n} = 0$, Similarly $\lim_{x \to -\infty} \frac{c}{x^n} = 0$

eg: $\lim_{x \to \infty} \dfrac{5x^2+3}{8x^2-7x+2}$ and $\lim_{x \to -\infty} \dfrac{5x^2+3}{8x^2-7x+2}$

try make use of the fact that $\lim_{x \to \pm\infty} \dfrac{c}{x^n} = 0$.

by dividing the Numerator & Denominator by degree of either Numerator or Denominator

Degree of Num. = Degree of Den. = 2

Divide with x^2

$$\lim_{x \to \infty} \frac{\frac{5x^2+3}{x^2}}{\frac{8x^2-7x+2}{x^2}} = \lim_{x \to \infty} \frac{5 + \cancelto{0}{\frac{3}{x^2}}}{8 - \cancelto{}{\frac{7}{x}} + \cancelto{0}{\frac{2}{x^2}}} = \frac{5}{8}$$

$$\lim_{x \to -\infty} \frac{\frac{5x^2+3}{x^2}}{\frac{8x^2-7x+2}{x^2}} = \lim_{x \to \infty} \frac{5 + \cancelto{0}{\frac{3}{x^2}}}{8 - \cancelto{}{\frac{7}{x}} + \cancelto{0}{\frac{2}{x^2}}} = \frac{5}{8}$$

ONLY one H.A

$y = \frac{5}{8}$

eg #

$$\lim_{x \to \infty} \frac{x+8}{x^2-11} \quad \text{and} \quad \lim_{x \to -\infty} \frac{x+8}{x^2-11}$$

Degree of Num. = 1

Degree of Den = 2

Divide with x^2

$$\lim_{x \to \infty} \frac{\frac{x+8}{x^2}}{\frac{x^2-11}{x^2}} = \lim_{x \to \infty} \frac{\cancel{\frac{1}{x}}^{\,0} + \cancel{\frac{8}{x^2}}^{\,0}}{1 - \cancel{\frac{11}{x^2}}^{\,0}} = 0$$

Only one H.A

$y = 0$

Similarly!

$$\lim_{x \to -\infty} \frac{x+8}{x^2-11} = 0$$

eg) $\lim_{x \to \infty} \frac{x^3+8}{x^2-11}$ and $\lim_{x \to -\infty} \frac{x^3+8}{x^2-11}$

degree of Num = 3

degree of Den = 2

Divide with x^2

$$\lim_{x \to \infty} \frac{\frac{x^3+8}{x^2}}{\frac{x^3-11}{x^2}} = \lim_{x \to \infty} \frac{x + \frac{8}{x^2}}{1 - \frac{11}{x^2}} = \lim_{x \to +\infty} x = +\infty$$

0

0

Similarly

$$\lim_{x \to -\infty} \frac{x^3-8}{x^2-11} = \lim_{x \to -\infty} x = -\infty$$

$\boxed{\text{NO H.A}}$

When $\lim_{x \to \pm\infty} f(x) = +\infty$ or $-\infty$

$f(x)$ can have a slant Asymptote

eg: $\dfrac{x^3+8}{x^2-11}$

① Divide by degree of denominator

$$\frac{x^3+8}{x^2-11} = \frac{x + \cancel{\frac{8}{x^2}}\to 0}{1 - \cancel{\frac{11}{x^2}}\to 0}$$

$\Big\}$ $y = x$ is slant Asymptote.

② Long Division

$$x^2-11 \overline{\big)\, x^3 + 8} \quad \big(x$$
$$\underline{\overset{(-)}{x^3} \; \overset{(+)}{-\,11x}}$$
$$11x+8$$

$$\Rightarrow \frac{x^3+8}{x^2-11} = \underbrace{x}_{\text{Remaining as } x\to\infty} + \frac{11x+8}{x^2-11} \to 0$$

eg! #

$$\lim_{x \to +\infty} \frac{7x+2}{3x+\sqrt{4x^2+7}}$$

Divide with x

$$= \lim_{x \to +\infty} \frac{\frac{7x+2}{x}}{\frac{3x+\sqrt{4x^2+7}}{x}} = \lim_{x \to +\infty} \frac{7+\frac{2}{x}}{3+\frac{\sqrt{4x^2+7}}{x}}$$

use $x=\sqrt{x^2}$

$$= \lim_{x \to +\infty} \frac{7+\frac{2}{x}}{3+\sqrt{\frac{4x^2+7}{x^2}}}$$

$$= \lim_{x \to +\infty} \frac{7+\frac{2}{x}\to 0}{3+\sqrt{4+\frac{7}{x^2}\to 0}} = \frac{7}{5}$$

How about $\lim\limits_{x\to -\infty} \frac{7x+2}{3x+\sqrt{4x^2+7}}$?

is this the same as $\lim\limits_{x\to\infty} \frac{7x+2}{3x+\sqrt{4x^2+7}}$?

$$\lim_{x\to-\infty} \frac{7x+2}{3x+\sqrt{4x^2+7}} = \lim_{x\to-\infty} \frac{7+\frac{2}{x}}{3+\frac{\sqrt{4x^2+7}}{x}}$$

$x = -\sqrt{x^2}$, because x is -ve

$$= \lim_{x\to-\infty} \frac{7+\cancel{\frac{2}{x^2}}^{\,0}}{3-\sqrt{4+\cancel{\frac{7}{x^2}}^{\,0}}} = 7$$

$Y = \frac{7}{5}$ & $Y = 7$ are two H.A !!